TABLE 1

Properties of LMC Double-Mode RR Lyrae

α (J2000.0)	δ (J2000.0)	Identifier	N_V	$\langle V \rangle$ (mags)	N_R	$\langle R \rangle$ (mags)	P_0 (days)	P_1/P_0
05:14:33.2	−70:25:49	9..5482...957	225	19.25	228	19.11	0.4589520(025)	0.7420698(044)
05:21:33.5	−70:31:57	13..6691..4052	420	19.28	434	19.09	0.4608707(031)	0.7426381(061)
05:21:33.3	−70:31:57	6..6691..1003	281	19.35	284	19.23	0.4608670(036)	0.7426719(079)
05:19:24.3	−68:12:34	3..6363...626	449	19.39	456	19.16	0.4612340(017)	0.7429317(036)
05:32:41.3	−69:22:49	82..8523..1260	451	19.08	531	18.88	0.4612970(025)	0.7423234(053)
05:23:37.8	−70:32:40	13..7054..2970	403	19.15	202	18.95	0.4616550(025)	0.7430035(049)
05:32:24.9	−69:15:12	82..8404..1230	456	19.16	468	19.02	0.4630523(018)	0.7424662(041)
05:10:38.3	−68:24:42	2..4908..1321	220	19.63	233	19.42	0.4633006(026)	0.7425873(059)
05:49:06.8	−70:36:02	12.11167...644	486	19.46	489	19.20	0.4636594(020)	0.7431360(041)
05:10:43.0	−69:40:33	5..4889...921	249	19.37	273	19.27	0.4640494(030)	0.7429591(059)
05:16:09.3	−70:45:21	13..5720...646	184	19.04	191	18.86	0.4642093(035)	0.7432729(074)
05:31:46.3	−70:46:22	7..8381..2836	356	19.29	352	19.17	0.4647207(017)	0.7428002(033)
05:23:15.7	−70:30:27	6..6933...939	308	19.16	295	19.02	0.4648526(020)	0.7431166(039)
05:19:33.1	−69:18:04	80..6346..2144	372	18.95	218	18.86	0.4648986(016)	0.7428216(033)
05:46:33.0	−70:29:50	12.10684...733	499	19.31	515	19.03	0.4659660(025)	0.7432932(044)
05:09:16.9	−70:28:48	9..4635...878	254	19.10	254	19.04	0.4662452(031)	0.7429193(056)
05:19:55.7	−70:20:26	6..6452...607	141	19.09	194	18.88	0.4665101(051)	0.7428546(106)
05:13:49.8	−69:45:05	5..5372..1429	313	19.11	318	18.93	0.4668059(027)	0.7436038(053)
05:22:24.7	−68:41:55	3..6839..2145	452	19.36	485	19.16	0.4686444(023)	0.7426002(047)
05:38:55.6	−70:45:30	11..9471...659	360	18.91	354	18.69	0.4688101(040)	0.7431301(079)
04:49:26.5	−67:58:58	47..1526...209	197	19.07	197	18.90	0.4692676(046)	0.7439077(090)
05:23:55.0	−70:54:15	13..7048...418	185	19.13	188	18.99	0.4694934(068)	0.7429434(157)
05:08:00.5	−69:21:51	1..4530..1282	379	19.10	363	18.96	0.4696101(028)	0.7438903(049)
05:13:09.2	−69:40:46	5..5252..1096	255	19.31	258	19.05	0.4697376(032)	0.7433429(064)
04:50:40.2	−67:57:48	47..1647...290	191	19.30	205	19.13	0.4700310(034)	0.7438457(069)

TABLE 1—*Continued*

α (J2000.0)	δ	Identifier	N_V	⟨V⟩ (mags)	N_R	⟨R⟩ (mags)	P_0 (days)	P_1/P_0
05:16:30.8	−70:16:52	6..5848..1072	298	19.25	310	19.09	0.4700680(022)	0.7429545(043)
05:11:30.7	−69:27:28	5..5013..3494	284	19.34	298	19.20	0.4713201(050)	0.7435611(083)
05:24:11.3	−68:43:49	3..7081...943	498	19.18	467	19.04	0.4714000(049)	0.7439027(101)
05:08:48.6	−69:07:30	1..4655..2021	307	19.44	304	19.30	0.4725090(029)	0.7435483(053)
05:12:14.8	−70:20:17	9..5121...827	257	19.34	266	19.16	0.4730707(031)	0.7436929(061)
04:57:37.7	−68:54:06	18..2843..2547	189	19.27	199	19.04	0.4737667(033)	0.7437043(064)
05:09:55.0	−70:24:37	9..4757..2194	206	18.87	179	18.84	0.4739706(044)	0.7432945(093)
04:57:01.9	−68:49:44	18..2723...838	177	19.17	185	18.96	0.4739266(063)	0.7432322(123)
05:18:11.3	−71:12:19	13..6076...548	437	19.37	455	19.17	0.4741067(038)	0.7433912(069)
05:07:33.8	−69:54:51	10..4401..4272	196	19.35	211	19.18	0.4745026(056)	0.7439392(093)
05:25:17.1	−69:17:04	80..7315..1538	411	19.17	414	18.97	0.4750387(020)	0.7438703(039)
05:38:04.3	−70:45:17	11..9350...903	296	19.37	294	19.22	0.4757933(027)	0.7442452(046)
05:45:48.1	−70:44:06	12.10560...178	257	18.89	247	18.78	0.4758595(027)	0.7438943(052)
05:02:44.8	−68:09:59	19..3580...465	232	19.08	236	18.91	0.4775832(052)	0.7440243(093)
05:05:34.5	−69:13:24	1..4049..2260	356	19.06	380	18.84	0.4779170(036)	0.7441433(070)
05:26:39.8	−70:05:20	77..7545..1987	864	18.98	944	18.82	0.4781246(014)	0.7433261(029)
04:57:27.4	−69:08:26	18..2719...645	199	19.07	206	18.89	0.4782761(036)	0.7439030(063)
05:22:58.8	−70:59:54	13..6926...568	233	19.12	234	18.99	0.4788612(027)	0.7431244(058)
05:11:25.5	−69:51:32	5..5007..1447	307	18.86	309	18.65	0.4792201(043)	0.7443146(073)
04:50:04.7	−67:55:22	47..1527...361	368	19.29	378	19.10	0.4804636(020)	0.7435512(039)
05:18:17.1	−70:55:16	13..6080...591	256	18.94	260	18.77	0.4808895(027)	0.7439394(055)
05:20:04.3	−68:21:41	3..6481...796	433	19.63	468	19.36	0.4815493(028)	0.7441826(048)
05:13:05.4	−68:20:36	2..5272...238	250	19.36	263	19.13	0.4815702(044)	0.7437836(081)
05:22:44.8	−70:36:35	6..6811...651	300	19.25	294	19.12	0.4833567(028)	0.7445694(054)
05:16:27.1	−71:02:36	13..5836...525	462	19.02	271	18.85	0.4840640(033)	0.7445305(059)
05:20:14.9	−70:27:05	6..6450...903	309	19.11	309	19.01	0.4859942(025)	0.7443086(048)
05:14:11.7	−69:58:35	5..5489..1289	276	19.27	273	19.08	0.4862676(043)	0.7443577(067)
05:04:36.0	−69:24:18	1..3925..1407	376	18.84	384	18.70	0.4899693(040)	0.7446839(066)
05:24:16.3	−69:08:13	80..7075...796	415	19.12	420	18.96	0.4904683(020)	0.7447098(035)
05:25:42.4	−69:09:55	80..7316..1285	395	19.20	416	19.02	0.4910994(017)	0.7442975(033)

TABLE 1—*Continued*

α	δ (J2000.0)	Identifier	N_V	$\langle V \rangle$ *(mags)*	N_R	$\langle R \rangle$ *(mags)*	P_0 *(days)*	P_1/P_0
05:30:16.4	−68:43:44	82..8049...898	507	19.42	497	19.20	0.4919567(025)	0.7443936(051)
05:24:40.6	−68:41:07	3..7203...866	456	19.11	414	18.89	0.4930655(027)	0.7442035(052)
05:06:12.4	−69:50:30	10..4160..1090	200	19.26	213	19.02	0.4935916(044)	0.7445237(079)
05:26:33.6	−68:56:54	80..7441...960	771	19.36	384	19.08	0.4958965(024)	0.7439956(041)
05:16:45.1	−70:57:08	13..5838...497	248	19.08	146	18.92	0.4967609(045)	0.7444112(080)
05:17:00.8	−70:05:42	6..5971..1408	240	19.41	242	19.23	0.4985900(038)	0.7446263(070)
05:17:37.1	−71:00:28	13..5958...518	227	19.21	233	19.02	0.4986095(048)	0.7446724(079)
05:38:40.5	−70:46:52	11..9470...676	341	19.21	341	19.02	0.4986304(038)	0.7445930(063)
05:22:23.1	−68:57:17	80..6836..1492	330	19.65	364	19.38	0.4999572(039)	0.7442186(061)
05:30:29.6	−70:40:23	7..8141..1044	367	19.11	371	18.97	0.5008413(041)	0.7444443(068)
05:05:41.2	−68:05:18	19..4066...382	240	18.83	239	18.64	0.5115859(044)	0.7444374(063)
05:09:37.6	−69:44:50	5..4767...952	298	18.94	292	18.74	0.5118882(030)	0.7446402(051)
05:22:35.4	−70:38:28	6..6810...428	311	19.08	315	18.90	0.5115917(035)	0.7455403(056)
05:22:35.3	−70:38:29	13..6810..2845	250	19.00	115	18.76	0.5115875(043)	0.7455497(065)
05:20:36.5	−68:15:49	3..6483...522	249	19.08	262	18.91	0.5136604(032)	0.7459381(050)
05:09:13.8	−68:13:47	2..4668...539	280	19.10	274	18.93	0.5150211(059)	0.7460380(093)
05:33:47.2	−70:22:37	11..8629..1482	314	19.14	321	18.93	0.5184966(039)	0.7447387(070)
05:28:00.6	−70:27:03	7..7660..1051	260	19.12	266	18.95	0.5250907(035)	0.7451420(063)
05:26:26.1	−69:03:40	80..7439..1826	337	19.48	177	19.16	0.5287074(043)	0.7459268(067)
05:46:21.3	−71:25:23	15.10671...500	245	19.10	253	18.85	0.5444242(047)	0.7461122(072)

The MACHO Project LMC Variable Star Inventory IV: Multimode RR Lyrae Stars, Distance to the LMC and Age of the Oldest Stars

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ABSTRACT

We report the discovery of 73 double-mode RR Lyrae (RRd) stars in fields near the bar of the LMC. The stars are detected among the MACHO database of short-period variables that currently contains about 7900 RR Lyrae stars. Fundamental periods (P_0) for these stars are found in the range 0.46-0.55 days and first overtone-to-fundamental period ratios are found to be in the range $0.742 < P_1/P_0 < 0.748$. A significant fraction of our current sample have period ratios smaller than any previously discovered RRd variables. We present mean magnitudes, colors, and lightcurve properties for all LMC RRd stars detected to date. The range in period ratios is unexpectedly large.

We present a determination of absolute magnitudes for these stars based primarily on pulsation theory and the assumption that all observed stars are at the fundamental blue edge (FBE) of the instability strip. Comparison of the calibrated MACHO V and R_{KC} photometry with these derived absolute magnitudes yields an absorption-corrected distance modulus to the LMC of 18.57 ± 0.19 mag which is in good agreement with that found (18.5) through comparison of galactic and LMC Cepheids.

Adopting this luminosity calibration, we derive an increase in the distance modulus, and thus a reduction in the age found via isochrone fitting for M15 of about 33% and discuss the implications for cosmology.

Subject headings: stars: RR Lyrae : LMC

1. Introduction

Multimode RR Lyrae, also known as RRd type, are especially interesting tests of pulsation and evolutionary theory because precise period ratios allow an estimate of the mass of each star. This paper reports properties of a total of 73 such stars in the Large Magellanic Cloud (LMC), nearly twice as large as the number of all known RRd stars prior to this work.

The MACHO Project LMC RR Lyrae database is described in Alcock *et al.* (1996) where accumulated V and R photometry for approximately 7900 RR Lyrae stars in 22 intensively-observed fields has been analysed to determine the distributions of period and amplitude. In that paper, a random selection of 500 lightcurves was examined and two multimode stars were identified. The ratios of first overtone-to-fundamental period, P_1/P_0, of those stars were found to be lower than any previously discovered RRd stars.

This paper reports the discovery of a further 71 RRd stars. We first discuss the observational properties of this new sample. Next we estimate absolute magnitudes using pulsation theory. Finally, we note implications of the derived luminosities for cosmology.

2. Results

2.1. Photometry

The photometric reduction procedures for the MACHO images are described in Alcock *et al.* (1996). The MACHO instrumental magnitudes have been placed on the standard photometric system of Kron-Cousins V and R through comparison with Landolt (1992) standard stars that were transferred to the 22 high-priority MACHO fields. While gross zero-point differences between fields have been removed, small offsets within each field exist

as an artifact of the MACHO photometry reduction procedure. We conservatively adopt an error in V or R of ±0.10 mag. The transformation between MACHO instrumental color and Kron-Cousins (V-R) is less affected by the internal zero-point offsets, therefore, the color for each star in Table 1 has an estimated uncertainty of ±0.05 mag. We arrive at these estimates of the photometric uncertainties through numerous consistency checks within the MACHO database and by comparisons with published photometry of LMC stars.

The lightcurves used in this analysis represent all data acquired for these stars during an interval of approximately 1400 days and typically represent 400-700 points per color. In Figure 1 we show the mean apparent V and (V-R) for LMC double-mode RR Lyrae and illustrate their position relative to other LMC field stars. The lightcurves for these stars are available on the MACHO Project home pages at the URLs `http://wwwmacho.anu.edu.au/` and `http://wwwmacho.mcmaster.ca/` .

2.2. Selection Criteria

Double-mode RR Lyrae candidates were selected from our color-magnitude diagram-based RR Lyrae sample by two techniques. The first, described in Alcock *et al.* (1996), relied on visual examination of 500 lightcurves using the IRAF [13] PDM task. More recently, candidate RRd stars were selected based on their measure-of-fit to a singly-periodic lightcurve, their period and lightcurve amplitude.

The strong period dependence of our selection criteria deserves mention. For photometric periods between 0.35 and 0.40 days - corresponding to fundamental periods between 0.46 and 0.50 days - our selection criteria identify double-mode RR Lyrae very efficiently. Shortward and longward of this range, our yield is reduced by the increase

[13] Developed at the National Optical Astronomical Observatories

in false positives with period estimates of one-third and one-half of a day, respectively. The measure-of-fit statistic is less effective at long periods due to the presence of Blazhko variation in some RRab pulsators. Therefore, we caution the reader that our current sample is not complete and that the distribution in period (and therefore mass) is unrepresentative.

2.3. Observed Properties of the Sample

We summarize our discovery of LMC RRd stars in Table 1. This table lists, from left-to-right, the equinox J2000.0 right ascension and declination, the internal database identifier, the number of measurements and mean magnitude in V and R_{KC}, respectively, the estimate of the fundamental period P_0 (in days) and the period ratio, P_1/P_0, the semi-amplitude of the Fourier component for the frequency corresponding to P_1 (in mags), and the ratio of the amplitudes for the first Fourier components of the first-overtone and fundamental modes. The uncertainties in the period and period ratio are expressed in parentheses in units of the least significant digits. Table 1 contains 75 entries. The pairs (`13.6691.4052`, `6.6691.1003`) and (`13.6810.2845`, `6.6810.428`) represent the same stars found twice in overlapping regions of different MACHO fields. The total number of distinct stars is therefore 73. We remind the reader that our preferred nomenclature is MACHO* followed by the right ascension and declination as listed in Table 1, with no spaces.

The periods of the RR Lyrae were initially estimated using the IRAF PDM task as described in Alcock *et al.* (1996). The periods, period ratios and modal amplitudes reported in Table 1 are the result of a weighted least-squares fit to the RRd lightcurves for the first three Fourier terms in each mode and all coupling terms.

The ratio of first-overtone to fundamental mode amplitudes is shown in Figure 2. It is clear that this sample of stars is similar to known RRd's in that, almost without exception,

the amplitude of the first-overtone mode is greater than that of the fundamental. We note that the selection criteria used above are biased against stars with fundamental periods shorter than 0.46 days and that amplitude ratios smaller than unity may therefore be under-represented.

2.4. Absolute Magnitudes from Pulsation Theory

It is, in theory, possible to estimate absolute magnitudes of RRd stars by combining the Stefan-Boltzmann equation and the period-density relations. The result of such an exercise is an expression relating period to luminosity, effective temperature, and mass. Unlike singly-periodic RR Lyrae stars, the period ratio allows us to estimate the mass based on atmospheric pulsation models. Provided that we can determine the effective temperature, the set of required observables is complete. Such a relationship is well-known for Cepheid variables where the existence of the instability strip boundaries gives rise to an obvious period-luminosity relation. For double-mode RR Lyrae stars we expect a similar bounded region in the color-magnitude diagram which will also give rise to a period-luminosity relationship.

We will now proceed through the steps necessary to obtain absolute magnitude estimates. First, the estimation of masses from periods and period ratios will be described. Second, the estimation of effective temperature and the relationship of these stars with the fundamental blue edge (FBE) of the instability strip will be described. Third, we substitute for both effective temperature and mass in the pulsation equation and produce a period-luminosity relation. Finally, we estimate the errors introduced at each step.

2.4.1. Period Ratios and Masses

We show in Figure 3, the Petersen (1973) diagram of P_1/P_0 plotted against P_0 for stars in M15 (Nemec, 1995a), Draco (Nemec, 1995b), IC 4499 (Clement *et al.*, 1986, Walker and Nemec, 1996) and the LMC. We additionally mark on the diagram the locus of period ratios for model pulsators computed by Cox (1991) using OPAL91 opacities.

The distribution of the stars in Figure 3 shows that the LMC stars include a low mass extension to the multimode stars discovered in other systems. From Figure 3 it can be shown that a relation of the form

$$\log P_0 = 0.452 \log \mathcal{M} - 0.228 \tag{1}$$

applies across the indicated mass range of the multimode stars in the LMC and the Galactic Oosterhoff groups I and II.

The shortest period of Bailey type a,b stars among the total sample of LMC RR Lyrae stars is 0.459 days and, additionally, with a period ratio of about 0.7421 the overtone periods are close to one-third of a day and are thus more difficult to discover due to aliasing. We mark these period regions as hatched areas in Figure 3 to emphasize the degree to which the period-distribution of the multimode stars is affected by selection biases in this work. Nevertheless, it is clear that the tight locus of observed period and period-ratio in Figure 3 for RRd stars born in a wide range of environments is consistent with a tight relationship between luminosity, effective temperature and mass. In the following discussion we shall assign masses to the LMC stars on the basis of their position in the Petersen diagram relative to the Cox (1991) models. We note that there are potential uncertainties in these masses due to the lack of specific data on composition of the LMC stars. However. the abundance distribution of the LMC field RR Lyrae stars as derived from spectra does not deviate strongly from that of the galactic halo (Alcock *et al.* 1996).

2.4.2. Pulsation Models

The pulsation equation for model envelopes of RR Lyrae stars has been characterized using OPAL95 opacities by Bono *et al.* (1996) as

$$\log P_{0,tr} = 11.627 + 0.823 \log L_{tr} - 3.506 \log T_{e,tr} - 0.582 \log \mathcal{M}_{tr} \tag{2}$$

over the mass range 0.65-0.75 $M_{\odot}$. We have annotated the variables to reflect the assumption that the multimode stars are in the transition zone (the region between the fundamental blue edge and the first-overtone red edge) of the instability strip. Their equation 2 is a linear fit to the results of period determinations for RR Lyrae star models using the most recent composition parameters in a study of the non-linear pulsation properties. The fit to the pulsation equation is in the spirit of the influential study of RR Lyrae star pulsation by van Albada and Baker (1971) and we note in passing that use of their form of the pulsation equation does not affect the results and conclusions drawn in this paper.

We make the further assumption (see, for example, Caputo 1990, Bono and Stellingwerf 1994) that there is close similarity between the temperature of the blue edge, (FBE) of the fundamental instability strip and the transition zone occupied by the multimode stars. P_{tr} is thus taken as the period of the fundamental mode at the blue edge of the instability strip. In the range of P_{tr} of the beat RR Lyrae stars, $T_{e,tr}$ varies from 6900 K to 7060 K in excellent agreement with the calculations of the fundamental and first-overtone instability strip boundaries (and the zone where both fundamental and first overtone are unstable), by Bono and Stellingwerf (1994). Their calculations show that the temperature of the strip in which pulsation in either the fundamental or first-overtone mode is allowed is relatively insensitive to mass or luminosity and is centered near 6840 K for the mass range 0.58-0.75 $M_{\odot}$ and luminosity range 1.6-1.9 $\log L_{\odot}$. The assumption that the double-mode stars lie near this temperature is also supported by the close similarity of the periods of the shortest period LMC Bailey type a,b stars at $P_0 = 0.457$ days to the average

period of the multimode stars at $P_0 = 0.48$ days.

For the FBE, we can derive a relationship between the fundamental period and the effective temperature by combining equation 8 of Sandage (1993a) – the relation between the shortest period for RRab stars and [Fe/H] – and equation 5 of Sandage (1993b) – the relation between $\log T_e$ and [Fe/H] for RRab stars – to obtain:

$$\log T_{e,tr} = 3.816 - 0.0984 \log P_{tr}. \tag{3}$$

We see that the FBE is very nearly vertical in the $\log L$, $\log T_e$ plane. Sandage expresses both the FBE period and temperature in terms of abundance [Fe/H] but since the range of [Fe/H] in the LMC field RR Lyrae stars is the same as in the calibrating galactic RR Lyrae clusters and field, we have written the FBE temperature directly in terms of period. The luminosity of the multimode stars is then

$$\log L_{tr} = 2.483 + 2.360 \log P_{tr} \tag{4}$$

Over the range of the transition periods covering the majority of the LMC multimode stars shown in Figure 3, at $P_0 = 0.46$ days, $\log L_{tr} = 1.69$ and at $P_0 = 0.52$ days, $\log L_{tr} = 1.81$, corresponding to absolute visual magnitudes of $M_V = +0.49$ and $+0.19$ respectively when the small bolometric corrections of Kurucz (1979) are applied. For the longer fundamental periods of the multimode stars in the metal-weak galactic globular cluster M 15 with a characteristic $P_0 = 0.54$ days, the derived luminosity corresponds to $M_V = +0.05$ mag. Most other calibrations of RR Lyrae star luminosities have been expressed in terms of composition and there has been considerable debate about the slope of the coefficient of the term in [Fe/H]. We note that if we adopt the composition value of [Fe/H] = -2.12 for M15 (Buonanno *et al.* 1989), our calibration has a roughly similar dependence on [Fe/H] to that discussed by Sandage (1993a,b,c) but caution that direct comparison of these dependencies is limited by the small number of clusters that contain multimode stars. We

also note that our calibration depends on the theoretical pulsation equation which, as we will show in the next section, appears to be supported by the luminosities of the LMC RR Lyrae stars. Ours is a brighter calibration of RR Lyrae star luminosities than many previously derived. This calibration receives support from the independent study of the Bailey type c RR Lyrae stars by Simon and Clement (1993) who compared the Fourier phase parameters and periods of the first overtone pulsators with linear and hydrodynamic pulsation models to determine temperatures and a luminosity calibration. The result of their calibration applied to the Reticulum cluster of the LMC yields roughly the same distance modulus as we derive below for the field LMC stars. Their discussion of the galactic multimode stars as checks on their calibration of the RRc variables additionally supports the assumptions we made above concerning the location of the multimode instability strip in the luminosity-temperature plane.

In his recent monograph, Smith (1995) reviews at least 6 methods, both fully and semi-empirical, which yield luminosities roughly 0.4 mags fainter than those derived here. Conversely, all these methods would predict the LMC RR Lyrae stars to be too faint by the same amount. In this paper we adopt the view that the characteristics of RR Lyrae stars in the LMC and the Galaxy are properly comparable and explore the consequences of that comparison.

2.4.3. Error Analysis

The application of the luminosity calibration derived in the previous section to the photometry of our multimode RR Lyrae stars in the LMC necessitates a careful examination of the uncertainties. We adopt a mass uncertainty of $\pm 0.05\ \mathcal{M}_{\odot}$ (Kovacs *et al.* 1992) in our $\log P_0$, $\log \mathcal{M}$ relation. Assuming double-mode RR Lyrae stars lie at the FBE, the uncertainty in equation (3), our $\log T_{e,tr}$, $\log P_{tr}$ relation derived from Sandage (1993a,b), is

dominated by the uncertainty of the (B-V), $logT_{e,tr}$ relation used to define the temperatures of galactic RR Lyrae along the FBE. We return to the unpublished model atmosphere calculations of Bell as seen in Butler *et al.* (1978), the model atmospheres of Kurucz (1979), and the instability strip analysis of Bono *et al.* (1996), and by intercomparison adopt at the FBE an uncertainty of 0.01 in $\log T_{e,tr}$. Propagating the errors through to equation (4), we find that at a given period the uncertainty in $\log L_{tr}$ is ± 0.065 resulting in an uncertainty in M_V of ± 0.16 mag. We have assumed equation (4) is an exact representation and there is negligible error in these bolometric corrections.

2.4.4. The LMC Distance Modulus

In Figure 1 we show the multimode stars in the V,(V-R) color-magnitude diagram (CMD). We adopt the reddening vector with $\mathrm{R_V = A_V/E(V-R) = 5.0}$ (Bessell, 1996) which is illustrated in Figure 1. Also shown is the adopted photometric uncertainty for each point. The insert shows the multimode stars in relation to a representative MACHO LMC CMD. Using the temperatures from equation (3), for stars with $\log g = 2.6$, we adopt the (B-V), (V-R) relation (Bessell 1996),

$$(V-R) = 0.004 + 0.566(B-V) \tag{5}$$

with an uncertainty of $\pm$ 0.017 in the zero point. From Butler *et al.* (1978) we interpolate the T (our adopted temperature for the FBE), (B-V) relations between [Fe/H] = -1 and -2 for [Fe/H] = -1.7, a value appropriate for the LMC RR Lyrae (Alcock *et al.* 1996) and derive a $(V-R)_0$, $\log P_{tr}$ relation,

$$(V-R)_0 = 0.19 + 0.15 \log P_{tr} \tag{6}$$

with an uncertainty in the zero-point of ± 0.02 mag derived from the uncertainties in equations (3) and (5) as discussed previously. Adopting our FBE temperature, the intrinsic

color, $(V-R)_0$, ranges from 0.14 to 0.15 mag for our multimode stars. Furthermore, we note that the mean reddening, $< E(V-R) > = 0.049$ mag, or $< E(B-V) > = 0.086$ mag, agrees well with other measurements of the reddening toward the LMC (Bessell 1991). In Figure 4 we show the de-reddened V magnitudes plotted against P_0. The error bars noted are derived from the uncertainty in V and in (V-R) scaled by the ratio of total to selective absorption ($R_V = 5$), equivalent to ± 0.27 mag. The contribution of the uncertainty in our adopted intrinsic color is systematic and we have not included it in the uncertainty of each individual de-reddened magnitude. Our multimode RR Lyrae have a mean period $< P_0 >$ $= 0.482$ corresponding to $< M_V > = 0.37 \pm 0.16$ mag. The mean de-reddened magnitude of our multimode RR Lyrae sample is $< V_0 > = 18.94 \pm 0.03$ mag, in excellent agreement with the mean de-reddened V magnitude of nearly 180 RR Lyrae found in LMC clusters (Walker 1992). The distance modulus of the LMC naturally follows: $(m-M)_{LMC} = 18.57$ $\pm$ 0.19 mag, where the final error is a combination of the uncertainty in $(V-R)_0$ (scaled by $R_V = 5$), the uncertainty in $< V_0 >$, and the uncertainty in $< M_V >$. Fitting the data of Figure 4 to the M_V, $\log P_{tr}$ calibration with a fixed slope yields the same LMC distance modulus. The best fit distance modulus is shown as a horizontal line in Figure 4. The corresponding M_V, $\log P_{e,tr}$, calibration is also shown. The distance modulus for the LMC multimode RR Lyrae stars derived here is found to be consistent with that (18.5 ± 0.1 mag) derived from the galactic calibration of classical Cepheids discussed by Walker (1992).

3. Discussion

Walker (1989,1992) has emphasized the apparent discrepancy of 0.3 magnitudes between RR Lyrae star luminosities derived from Baade-Wesselink analyses of galactic stars and the luminosities of LMC cluster variables if the Cepheid distance calibration is correct. We have shown here that there is a consistency between the luminosities derived from

the pulsation equation, and independent luminosity measures such as the LMC Cepheid distance calibration and the SN1987A ring distance (Gould 1995). Consequently, the pulsation properties of the multimode RR Lyrae stars appear to be a useful tool in distance studies through their calibration of the more common and easily discovered RRab stars.

The unusually large range in P_1/P_0 and the range in implied masses appear to present new questions about the evolution from the ZAHB. The Lee and Demarque (1990) and Sweigert (1987) HB evolutionary tracks do not carry the evolution of stars with masses as low as $0.55M_{\odot}$ as far as the instability strip but it does not appear that such stars could be of similar luminosity to the HB when they reach the instability strip following redward evolution. Cox (1991) goes further and states that stars with mass of $0.55M_{\odot}$ would never reach the instability strip at all. However as we see in Figure 3, there do exist LMC multimode RR Lyrae stars with this mass as derived from pulsation theory. In Alcock *et al.* (1996), we argued that the field HB of the LMC has a red morphology and that as a result, the beat RR Lyrae stars are rare. Our more recent detections still show fewer of these stars among the LMC field population than, say in the OoII cluster M15. We remind the reader that our survey is incomplete and that this fraction may rise. In the same paper, we also argued that the LMC field RR Lyrae star abundance distribution was similar to the halo galactic distribution with a modal [Fe/H] $= -1.7$. These findings led us to conclude that the LMC field was younger than most galactic globular clusters.

An implication of Figure 4 and the period-luminosity relation contained in equation (4) is a differential rescaling of the distances to galactic globular clusters (and the galactocentric distance) when HB luminosities are the criteria. Oosterhoff group II clusters are the most affected.

Walker (1992), in an important paper, supposed that the distance to the LMC was correctly defined by the galactic Cepheid distance scale and inferred that the LMC RR

Lyrae stars were 0.3 mag brighter than previously suggested. He examined the consequences of a global increase in HB star luminosities on the age and distance of galactic globular clusters. In this paper we have determined the RR Lyrae star luminosities to be significantly brighter than previously thought and have shown that this revision is most important for the OoII clusters in the Galaxy. For illustrative purposes we discuss the archetypal Galactic cluster M15 which has multimode RR Lyrae stars with a mean fundamental period of P_0 = 0.54 days. Equation (4) implies the HB $M_V = +0.05$ mag. Buonanno *et al.* (1989) list the brightness difference between MSTO and HB as 3.54 mag for this cluster, leading to $M_{V,to} = 3.59$ mag. These authors extract from evolutionary calculations the relation

$$\log t_9 = -0.41 + 0.37 M_{V,to} - 0.43Y - 0.13[Fe/H] \tag{7}$$

between age, turn-off luminosity and composition. For M15 we take Y = 0.23 and [Fe/H] = -2.15 (see Buonanno *et al.* 1989)). This equation yields an evolutionary age of 12.6 Gyr. (This estimate does not include the effect of any alpha-particle enrichment.) Minniti *et al.* (1996) have found [O/Fe] = 0.45 for M15 and we estimate that this results in a reduction in age of 0.6 Gyr based on the work of Salaris *et al.* (1993). The age estimate of 12.0 Gyr has the characteristic uncertainty of ±1.5 Gyr associated with MSTO ages. These age estimates can undoubtedly be refined by the fitting of evolutionary tracks to the recalibrated CMD's. We note that compared to a HB absolute magnitude of $M_V = +0.49$ mag for M15 (Armandroff 1989) our calibration produces a reduction in age of 33% which is relatively independent of the assumed value of alpha-element enrichment.

A fuller exploration of the parameter space of cosmological models defined by the age of the oldest stars is beyond the scope of this paper but we note that a 33% reduction in the estimated age of the oldest stars will help reconcile globular cluster ages with recent estimates of the Hubble constant, H_0.

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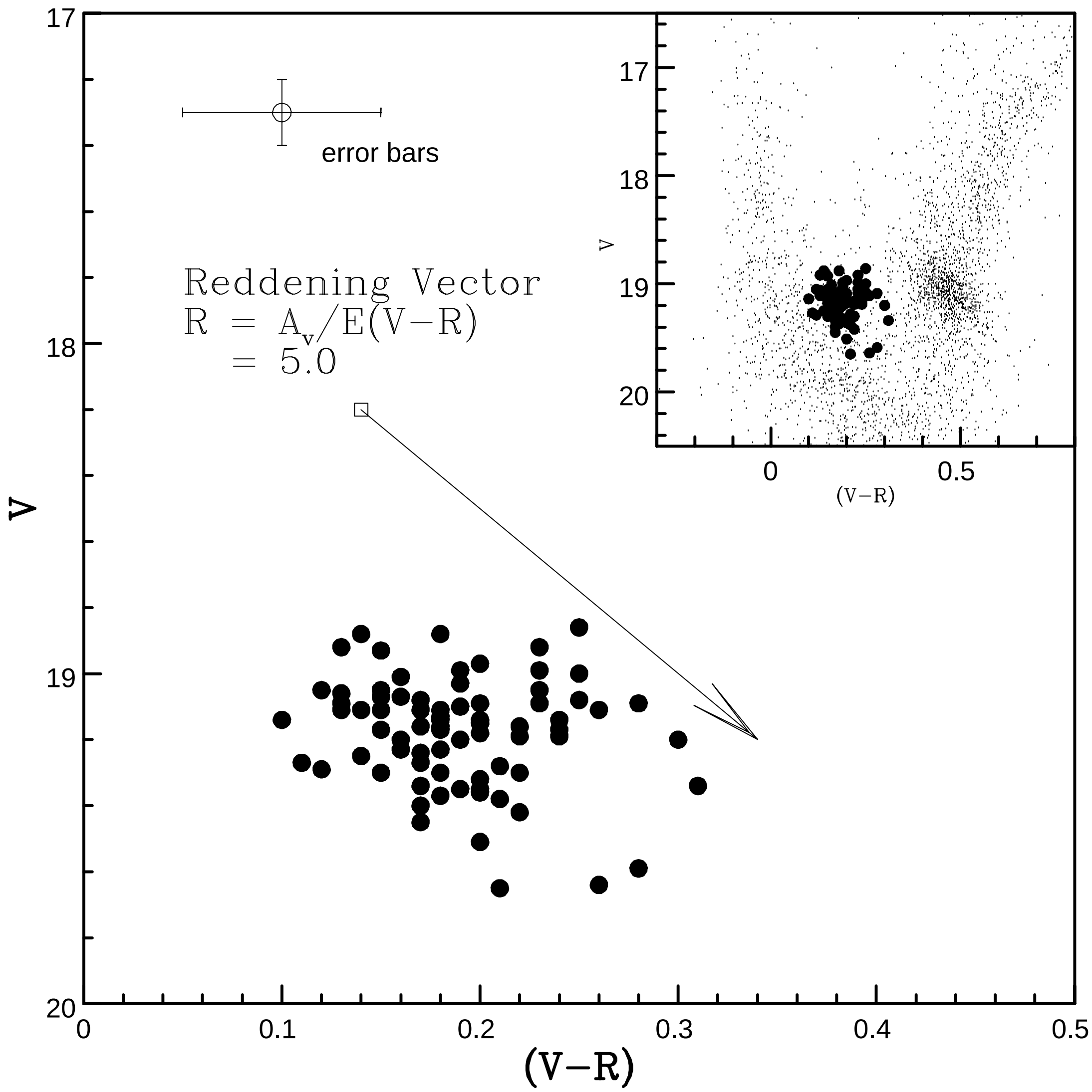

Fig. 1.— Observed median V magnitudes are plotted against derived (V-R) colors for the multimode RR Lyrae stars. The reddening vector is marked with an arrow. The insert shows the CMD of the multimode stars (filled circles), in relation to a representative CMD in an LMC field.

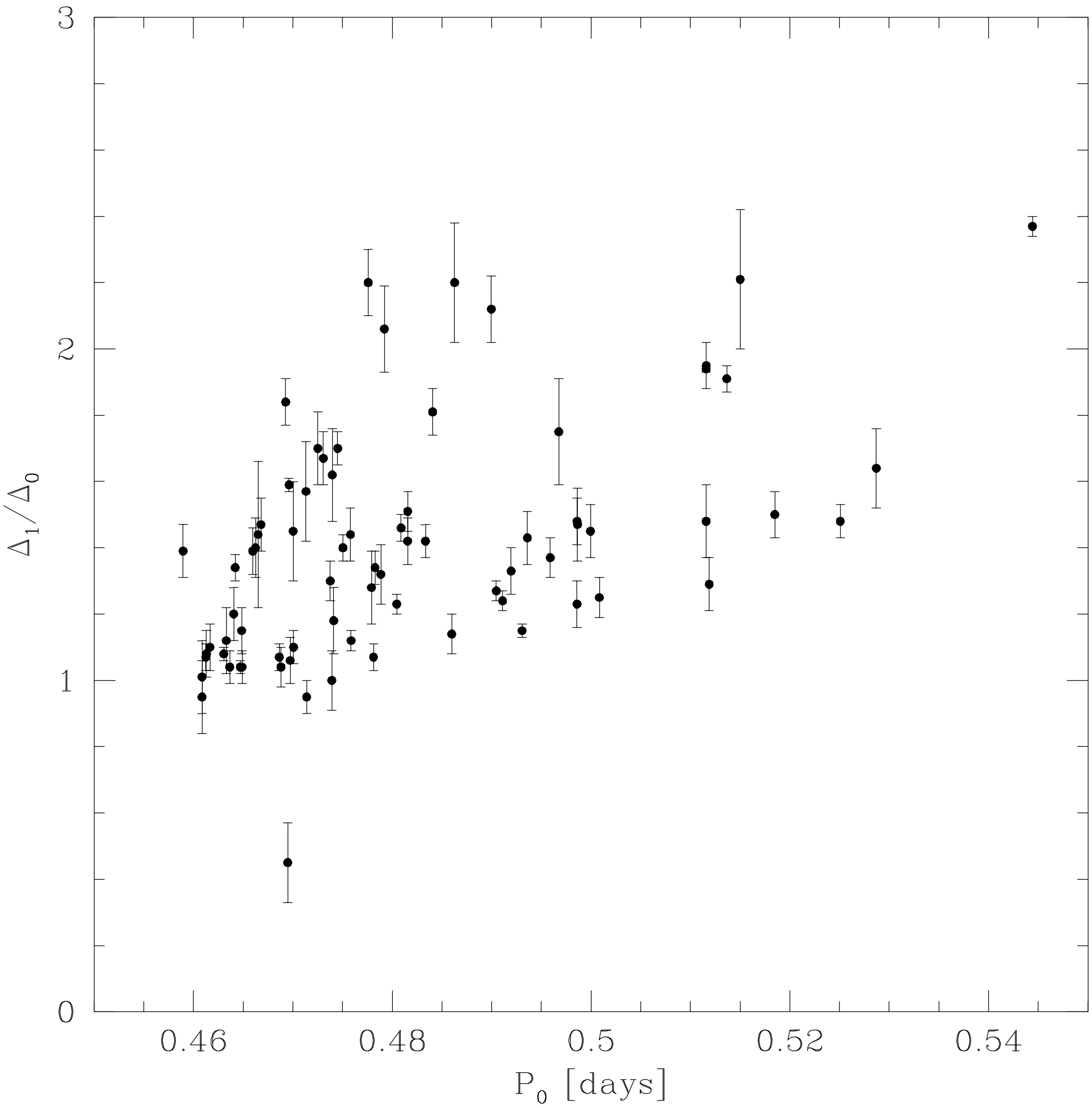

Fig. 2.— The ratio of the amplitudes of the first Fourier components for the first-overtone and fundamental modes, Δ_1/Δ_0, are plotted against the fundamental mode period P_0. Almost without exception, the amplitude of the first overtone mode is greater than that of the fundamental - a property that this sample shares with previously discovered RRd stars in globular clusters and dwarf spheroidal galaxies. We caution, however, that our sample is incomplete due to period-dependent selection biases and that more RRd with $\Delta_1/\Delta_0 < 1$ may yet be discovered in our LMC sample.

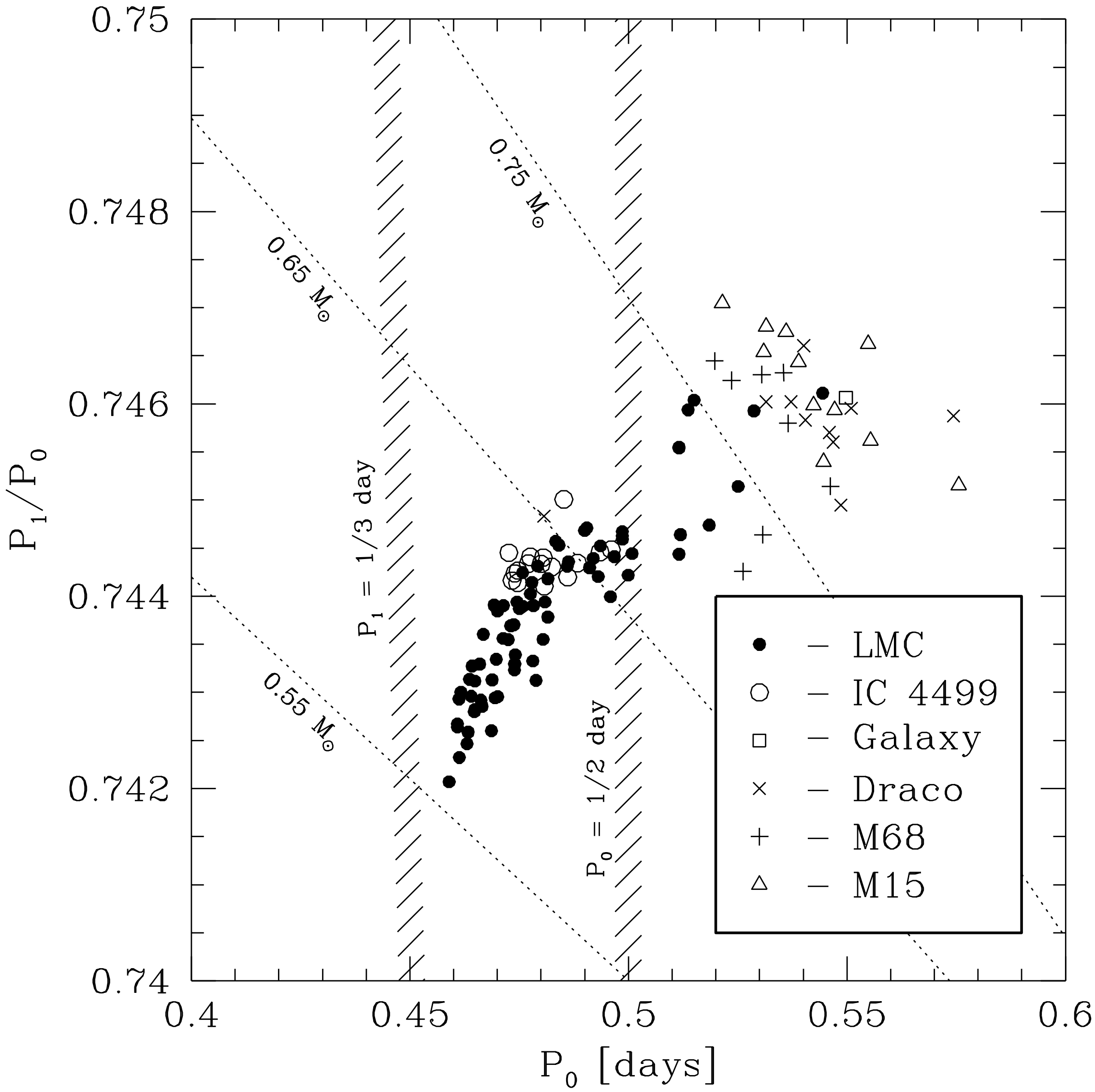

Fig. 3.— The Petersen diagram for multimode RR Lyrae stars where the first-overtone to fundamental period ratios P_1/P_0 are plotted against P_0. As described in the text, the LMC stars all have $P_0 < 0.5$ days while the longer period stars are from M15 and Draco. The loci of model pulsators calculated by Cox (1991) are shown for the masses of 0.55, 0.65 and 0.75 $\mathcal{M}_\odot$. The near-vertical line at $P_0 = 0.46$ days marks the location of the one-third day alias for the first-overtone period.

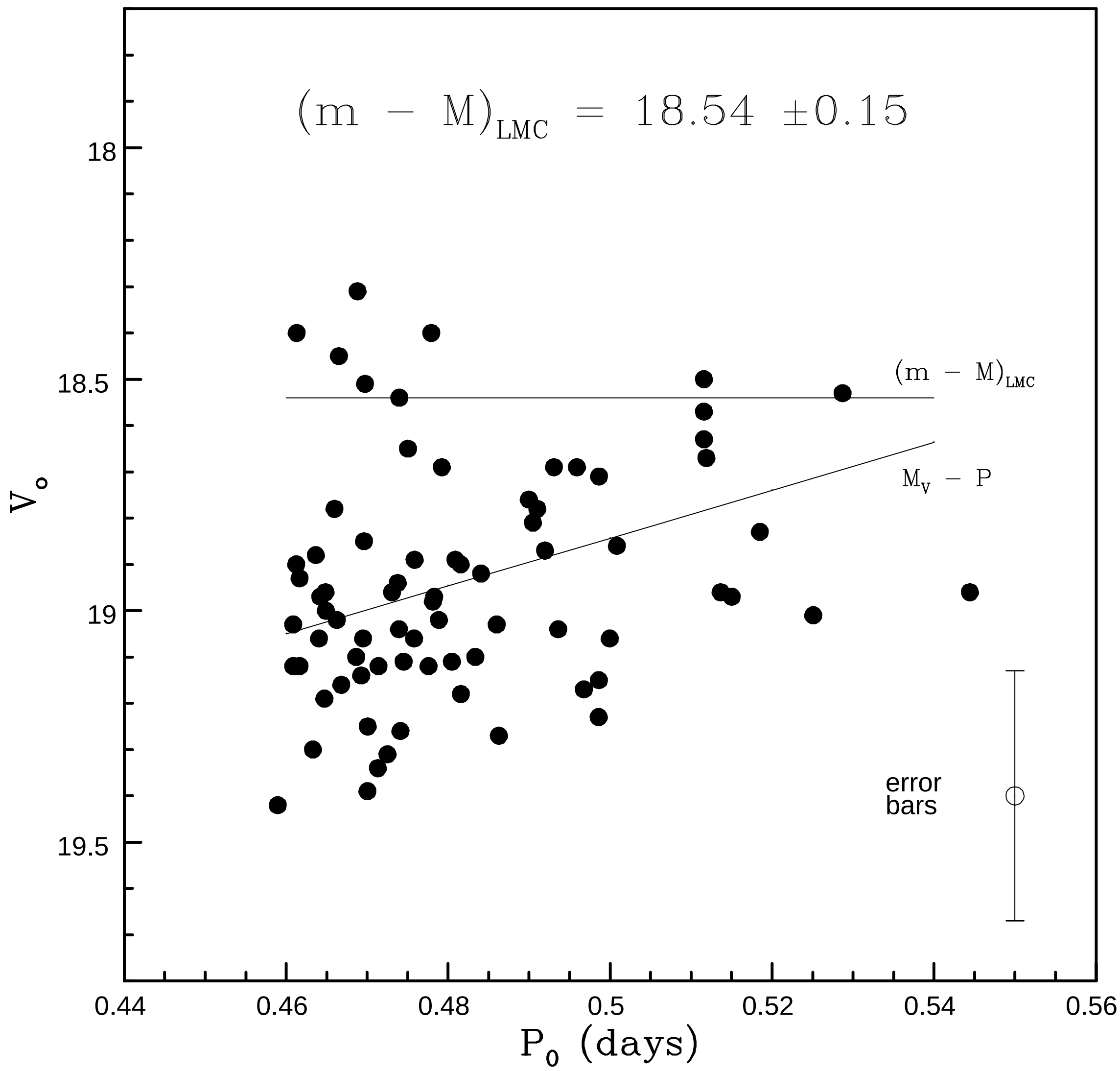

Fig. 4.— The absorption-corrected magnitudes, V_0, are plotted against the fundamental period P_0. The line has the slope given by the equation (4).